Exercise 5.9 (corrected)

Suppose x is $N(\mu_x, 1)$ and y is $N(\mu_y, 1)$, and they are independent. We are interested in the ratio $\theta = \mu_y/\mu_x$. Define $z \equiv y - \theta x$, so z is $N(0, 1+\theta^2)$, which depends only on θ and can be a basis for inference for θ. The so-called Fieller's CI is based on

$$P\left(\frac{(y-\theta x)^2}{1+\theta^2} < \chi^2_{1-\alpha}\right) = 1-\alpha.$$

Find the general conditions so that the 95% CI for θ is (i) an interval, (ii) two disjoint intervals, or (iii) the whole real line. Discuss how we should interpret part (iii). As a separate exercise, given $x = -1$ and $y = 1.5$,

(a) find Fieller's 95% CI for θ.

(b) plot the likelihood function of θ.

(c) find the $100(1-\alpha)$% CI for θ at various values of α, so you obtain the conditions that satisfy (i), (ii) or (iii) above. Explain the result in terms of the likelihood function.

(d) Discuss the application of confidence density concept to this problem.