LETTER TO THE EDITOR

Comment on: Confidence limits for the ratio of two rates based on likelihood scores: non-iterative method

by P. L. Graham, K. Mengersen and A. P. Morton,

From: Tang, Man-Lai Ng
Channing Laboratory
Department of Medicine
Brigham and Women’s Hospital
Harvard Medical School, Boston
MA 02115, U.S.A.

Hon Keung Tony
Department of Statistical Science
Southern Methodist University
Dallas
TX 75275-0332, U.S.A.

Graham, Mengersen and Morton studied the performance of three non-iterative confidence intervals for the ratio of two Poisson rates with respect to coverage probability. They are namely the Jaech’s interval, the score-based interval, and the Wald-based interval. The purpose of this letter is to point out that the aforementioned intervals are respectively identical to the square-root transformation interval (SR), the interval based on converting the Wilson’s interval for binomial parameter (B2), and the log-linear model Wald’s interval (LW) which were previously studied by Price and Bonett [1] in terms of mean coverage probability (MCP), mean width (MW), root mean squared error probability coverage about the nominal level (RMSE), and the proportion of the coverages that fell below the nominal level (PB). Most importantly, we would like to report in this letter some other observations than those reported by Graham et al. when other evaluation measures are considered in our coverage properties studies.

Let $X_0$ and $X_1$ be two independent Poisson variables with parameters $\lambda_0$ and $\lambda_1$, and amount of time or space sampled as the sample effort $n_0$ and $n_1$, respectively. Let $z_q$ be the upper $q$th percentile of the standard normal distribution. We first briefly review some 100(1−$\alpha$) per cent non-iterative confidence interval (CI) estimators, denoted as $[\phi_L, \phi_U]$, for the ratio $\phi = n_0 \lambda_1 / (n_1 \lambda_0)$.

A. Adjusted Wald-based method (AWM). Based on the statistic $\ln[(X_1 + 0.5)/(X_0 + 0.5)]$, Price and Bonett [1] derived the following Wald CI for $\phi$:

$$\phi_L = \left( \frac{n_0}{n_1} \right) \left( \frac{x_1 + 0.5}{x_0 + 0.5} \right) \exp \left[ -z_{\alpha/2} \sqrt{ \frac{1}{x_0 + 0.5} + \frac{1}{x_1 + 0.5} } \right]$$

$$\phi_U = \left( \frac{n_0}{n_1} \right) \left( \frac{x_1 + 0.5}{x_0 + 0.5} \right) \exp \left[ z_{\alpha/2} \sqrt{ \frac{1}{x_0 + 0.5} + \frac{1}{x_1 + 0.5} } \right]$$

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B. **Square-root transformation method (SRTM).** Price and Bonett [1] reported the following CI for \( \phi \) based on a square-root transformation of \( X_i \):

\[
\phi_L = \left( \frac{n_0}{n_1} \right)^2 \left[ \frac{\sqrt{(x_1 + 0.5)(x_0 + 0.5)} - 0.5z_{a/2}\sqrt{x_1 + x_0 + 1 - 0.25z_{a/2}^2}}{x_0 + 0.5 - 0.25z_{a/2}^2} \right]^2
\]

\[
\phi_U = \left( \frac{n_0}{n_1} \right)^2 \left[ \frac{\sqrt{(x_1 + 0.5)(x_0 + 0.5)} + 0.5z_{a/2}\sqrt{x_1 + x_0 + 1 - 0.25z_{a/2}^2}}{x_0 + 0.5 - 0.25z_{a/2}^2} \right]^2
\]

C. **Binomial Methods (BM).** Given \( X_0 + X_1 = x \). \( X_1 \) follows the binomial distribution \( B(x, \pi) \) with \( \pi = \lambda_1/(\lambda_0 + \lambda_1) \). Hence, for any CI for \( \pi \), say \( [\pi_L, \pi_U] \), a CI for \( \phi \) can be obtained as

\[
\phi_L = \frac{n_0(1 - \pi_U)}{n_1\pi_U}
\]

\[
\phi_U = \frac{n_0(1 - \pi_L)}{n_1\pi_L}
\]

Common choices for \( \pi_L \) and \( \pi_U \) include (a) **Clopper–Pearson Method (CPBM).** \( \pi_L = [1 + (x_1 + 1)/(x_0F_{1-\alpha/2,2x_0,2(x_1+1)})]^{-1} \) and \( \pi_U = [1 + x_1/((x_0 + 1)F_{3/2,2(x_0+1),2x_1})]^{-1} \) where \( F_{a,b,q} \) is the \( 1 - \alpha \) quantile from the \( F \) distribution with degrees of freedom \( a \) and \( b \). (b) **Wilson’s Method (WBM).** \( \pi_L = [x_1/(x_1 + z_{a/2}^2/x)] \left\{ \hat{\pi} + (z_{a/2}^2/2x_1) - z_{a/2}\sqrt{(1/x_1)(\hat{\pi}(1 - \hat{\pi}) + z_{a/2}^2/4x_1)} \right\} \) and \( \pi_U = [x_1/(x_1 + z_{a/2}^2/x)] \left\{ \hat{\pi} + (z_{a/2}^2/2x_1) + z_{a/2}\sqrt{(1/x_1)(\hat{\pi}(1 - \hat{\pi}) + z_{a/2}^2/4x_1)} \right\} \) with \( x = x_0 + x_1 \) and \( \hat{\pi} = x_0/x_1 \); and (c) **Agresti–Coul Method (ACBM).** \( \pi_L = \hat{\pi}_a - z_{a/2}\sqrt{\hat{\pi}_a(1 - \hat{\pi}_a)/x} \) and \( \pi_U = \hat{\pi}_a + z_{a/2}\sqrt{\hat{\pi}_a(1 - \hat{\pi}_a)/x} \) with \( \hat{\pi}_a = (x_0 + 2)/(x_0 + x_1 + 4) \).

Here, we observe that SRTM and AWM are, respectively, identical to the Jaceh’s interval and Wald’s interval (by adding 0.5 to the observed counts), which were discussed by Graham et al. Besides, it is important to note that WBM is equivalent to Graham et al.’s score method (denoted as GSM). To see this, we observe that the lower and upper limits of the 100(1 – \( \alpha \)) per cent CI based on GSM are given by

\[
\phi_{L, S} = \frac{n_0}{n_1} \left\{ \frac{2x_0x_1 + z_{a/2}^2x}{2x_0^2} \right\}
\]

\[
\phi_{U, S} = \frac{n_0}{n_1} \left\{ \frac{2x_0x_1 + z_{a/2}^2x + \sqrt{z_{a/2}^2x(4x_0x_1 + z_{a/2}^2x)}}{2x_0^2} \right\}
\]
Without loss of generality, let \( n_0 = n_1 \). Hence, we have

\[
2x_0^2 \phi_{L,S} = 2x_0x_1 + z_{2/2}^2 x - \sqrt{z_{2/2}^2 x (4x_0x_1 + z_{2/2}^2 x)}
\]

\[
2x_0^2 \phi_{U,S} = 2x_0x_1 + z_{2/2}^2 x + \sqrt{z_{2/2}^2 x (4x_0x_1 + z_{2/2}^2 x)}
\]

\[
\phi_{U,S} \phi_{L,S} = \left( \frac{x_1}{x_0} \right)^2
\]

Based on WBM, we can easily show that \( \pi_L = \left[ 2x_0x + z_{2/2}^2 x - \sqrt{z_{2/2}^2 x (4x_0x_1 + z_{2/2}^2 x)} \right] / [2x(x + z_{2/2}^2)] \), which implies \( \pi_L = [2x_0(1 + \phi_{L,S})]/[2x(x + z_{2/2}^2)] \). Further, we have \( 1 - \pi_L = (2x_0^2 + 2x_0^2 \phi_{U,S})/[2x(x + z_{2/2}^2)] \). Hence, we have \( \phi_U = (1 - \pi_L)/\pi_L = (x_1^2 + x_0^2 \phi_{U,S})/[x_0^2 (1 + \phi_{L,S})] = [(x_1/x_0)^2 + \phi_{U,S}]/(1 + \phi_{L,S}) = (\phi_{L,S} \phi_{U,S} + \phi_{U,S})/(1 + \phi_{L,S}) = \phi_{U,S} \) and similarly \( \phi_L = \phi_{L,S} \).

In the evaluation of the performance of various CI estimators, Graham et al.’s work was inadequate in several aspects. Firstly, their overlook of the work by Price and Bonett [1] leads to the exclusion of two potential competitors (i.e. CPBM and ACBM) and the failure of noticing the equivalence between WBM and their score-based method. Secondly, they merely considered coverage probability as their evaluation tool for CI performance. Obviously, MW, RMSE, and PB mentioned in the beginning of this letter are amongst other important indicators [1]. We will include the left-tail error rate (LTR) and right-tail error rate (RTER) in the subsequent re-evaluation. Thirdly, our results together with those from Price and Bonett [1] however lead to some other observations than those reported by Graham et al. Finally, neither Graham et al. nor Price and Bonett presented a rigorous definition of conservativeness, liberality and robustness in their CI estimator evaluation. We try to fill in these voids in the subsequent coverage properties studies.

Without loss of generality, we assume \( n_0 = n_1 \) in our coverage investigation. For small Poisson rates, we consider \( \lambda_0, \lambda_1 = 0.1 \) and these constitute \( N_c = 8000 \) different configurations. For moderate to large Poisson rates, we consider \( \lambda_0, \lambda_1 = 3(3)70 \) and \( N_c = 484 \) configurations are included. For SRTM, define \( \phi_L = 0 \) and \( \phi_U = \infty \) whenever \( x_0 + x_1 + 1 < 0.25z_{2/2}^2 \). For GSM, Graham et al. suggested setting \( x_0 = 0.5 \) whenever \( x_0 = 0 \). For all BMs (i.e. WBM, CPBM and ACBM), define \( \phi_L = 0 \) and \( \phi_U = \infty \) whenever \( x_0 = x_1 = 0 \); \( \phi_U = \infty \) whenever \( x_0 = 0 \) but \( x_1 > 0 \); and \( \phi_L = 0 \) whenever \( x_1 = 0 \) but \( x_0 > 0 \). For ACMB, set \( \phi_L \) to be \( \phi_L \) of WBM whenever \( \pi_U > 1 \); and \( \phi_U \) to be \( \phi_U \) of WBM whenever \( \phi_L \leq 0 \).

Given any configuration \( \lambda_0, \lambda_1 \), we consider the following criteria for CI evaluation.

1. Coverage probability. \( CP(\lambda_0, \lambda_1) = \sum_{x_0=0}^{\infty} \sum_{x_1=0}^{\infty} \left( e^{-\lambda_0} \lambda_0^{x_0} / x_0! \right) \left( e^{-\lambda_1} \lambda_1^{x_1} / x_1! \right) I_{\phi \in [\phi_L, \phi_U]}(x_0, x_1) \), where \( I_A(x_0, x_1) \) is the indicator function for event \( A \).
2. Left-tail error rate. \( LTER(\lambda_0, \lambda_1) = \sum_{x_0=0}^{\infty} \sum_{x_1=0}^{\infty} \left( e^{-\lambda_0} \lambda_0^{x_0} / x_0! \right) \left( e^{-\lambda_1} \lambda_1^{x_1} / x_1! \right) I_{\phi < \phi_U}(x_0, x_1) \).
3. Right-tail error rate. \( RTER(\lambda_0, \lambda_1) = \sum_{x_0=0}^{\infty} \sum_{x_1=0}^{\infty} \left( e^{-\lambda_0} \lambda_0^{x_0} / x_0! \right) \left( e^{-\lambda_1} \lambda_1^{x_1} / x_1! \right) I_{\phi > \phi_L}(x_0, x_1) \).
(4) Expected width. For moderate to large Poisson rates (i.e. \( \hat{\lambda}_0, \hat{\lambda}_1 = 7(3)70 \), \( \Pr(X_0 = 0) \) and \( \Pr(X_1 = 0) \) are negligible. In these cases, define expected width

\[
EW(\hat{\lambda}_0, \hat{\lambda}_1) = \sum_{x_0=0}^{\infty} \sum_{x_1=1}^{\infty} \frac{e^{-\hat{\lambda}_0} \hat{\lambda}_0^{x_0}}{x_0!} \frac{e^{-\hat{\lambda}_1} \hat{\lambda}_1^{x_1}}{x_1!} (\phi_U - \phi_L)
\]

For small Poisson rates (i.e. \( \hat{\lambda}_0, \hat{\lambda}_1 = 0.1(0.1)10 \)), both events \( \{X_0 = 0\} \) and \( \{X_1 = 0\} \) could happen with non-ignorable probabilities. However, when \( x_0 = 0 \) or \( x_1 = 0 \), the resultant CIs produce either infinite upper confidence limits or extraordinarily wide widths, and we can abandon these CIs for expected width calculation. In these situations, define (conditional) expected width

\[
EW(\hat{\lambda}_0, \hat{\lambda}_1) = \sum_{x_0=0}^{\infty} \sum_{x_1=1}^{\infty} \frac{e^{-\hat{\lambda}_0} \hat{\lambda}_0^{x_0}}{x_0!} \frac{e^{-\hat{\lambda}_1} \hat{\lambda}_1^{x_1}}{x_1!} (\phi_U - \phi_L) \frac{1 - \Pr(X_0 = 0) - \Pr(X_1 = 0) + \Pr(X_0 = X_1 = 0)}{\Pr(X_0 = 0) - \Pr(X_1 = 0) + \Pr(X_0 = X_1 = 0)}
\]

Before we continue our discussion, it should be noted that there is a duality between tests and confidence intervals. In fact, this duality relates the type I error rate of the test and the coverage probability of the corresponding interval. Cochran [2] defined that a test is robust if the actual type I error rate does not exceed 20 per cent of the nominal level. To refine his definition, we call a statistical test with an actual type I error rate exceeding the nominal level by more than 20 per cent a liberal test, a test with an actual type I error rate below the nominal level by more than 20 per cent a conservative test, and otherwise a robust test. In the language of confidence interval, a CI estimator is said to be conservative if \( CP(\hat{\lambda}_0, \hat{\lambda}_1) \geq 1 - (1 - 0.20)\alpha \), liberal if \( CP(\hat{\lambda}_0, \hat{\lambda}_1) \leq 1 - (1 + 0.20)\alpha \), robust otherwise.

(5) Index for conservativeness

\[
IC(\hat{\lambda}_0, \hat{\lambda}_1) = \begin{cases} 
1 & \text{if } CP(\hat{\lambda}_0, \hat{\lambda}_1) \geq 1 - (1 - 0.20)\alpha \\
0 & \text{if } CP(\hat{\lambda}_0, \hat{\lambda}_1) < 1 - (1 - 0.20)\alpha 
\end{cases}
\]

and

(6) Index for liberality

\[
IL(\hat{\lambda}_0, \hat{\lambda}_1) = \begin{cases} 
1 & \text{if } CP(\hat{\lambda}_0, \hat{\lambda}_1) \leq 1 - (1 + 0.20)\alpha \\
0 & \text{if } CP(\hat{\lambda}_0, \hat{\lambda}_1) > 1 - (1 + 0.20)\alpha 
\end{cases}
\]

In Tables I and II, we summarize our findings in terms of (a) Mean coverage probability (MCP). MCP = \( \sum_{\hat{\lambda}_0} \sum_{\hat{\lambda}_1} CP(\hat{\lambda}_0, \hat{\lambda}_1) / N_c \); (b) Minimum coverage probability (MinCP). MinCP = \( \min_{\hat{\lambda}_0, \hat{\lambda}_1} CP(\hat{\lambda}_0, \hat{\lambda}_1) \); (c) Mean expect width (MEW). MEW = \( \sum_{\hat{\lambda}_0} \sum_{\hat{\lambda}_1} EW(\hat{\lambda}_0, \hat{\lambda}_1) / N_c \); (d) Mean left-tail error rate (MLTER). MLTER = \( \sum_{\hat{\lambda}_0} \sum_{\hat{\lambda}_1} LTER(\hat{\lambda}_0, \hat{\lambda}_1) / N_c \); (e) Mean right-tail error rate (MRTER). MRTER = \( \sum_{\hat{\lambda}_0} \sum_{\hat{\lambda}_1} RTER(\hat{\lambda}_0, \hat{\lambda}_1) / N_c \); (f) Proportion of conservativeness (PC). PC = \( \sum_{\hat{\lambda}_0} \sum_{\hat{\lambda}_1} IC(\hat{\lambda}_0, \hat{\lambda}_1) / N_c \); and (g) Proportion of liberality (PL). PL = \( \sum_{\hat{\lambda}_0} \sum_{\hat{\lambda}_1} IL(\hat{\lambda}_0, \hat{\lambda}_1) / N_c \).

Firstly, as expected, CPBM is conservative and possesses the longest expected width especially for small Poisson rates. SRTM also produces wide expected width and could end up with extraordinarily small coverage probability (e.g. 9.37 per cent with true confidence level being 99 per cent) for small Poisson rates. Secondly, although WBM and GSM are shown to
be identical, the way the two methods handle the boundary cases (i.e. \( x_0 = 0 \) or \( x_1 = 0 \)) could greatly affect the resultant coverage properties. Here, WBM is more preferable because it has (i) smaller MCPs which are closer to the prespecified confidence levels; (ii) larger MinCP; (iii) symmetric left and right tail error rates; and (iv) smaller PC. It is noteworthy that GSM is the only method which produces non-symmetric tail error rates for small Poisson rates. Thirdly, disagreeing with Graham et al. we however find that AWM (Wald’s method with 0.5 adding to the observed counts) is generally more favorable than WBM or GSM. In this case, AWM not only possesses larger MCP and PC but also smaller MEW. In other words, AWM produces large coverage CIs with shorter expected widths. Graham et al. found AWM less desirable (conservative) simply because they did not take expected width into consideration. Besides, when the prespecified confidence level is large (e.g. 99 per cent) WBM and GSM could generate high proportion of liberal CIs. Fourthly, ACBM can be considered as a promising alternative in small confidence levels (e.g. \(<90\) per cent). In these cases, ACBM
Table II. Exact coverage properties (in per cent) of various intervals for $\phi$ over a finite grid of values $\lambda_0, \lambda_1 = 7(3)70$.

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<th>MEW</th>
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<td>WBM</td>
<td>99.03</td>
<td>98.96</td>
<td>2.97</td>
<td>0.49</td>
<td>0.49</td>
<td>0.21</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>GSM</td>
<td>99.03</td>
<td>98.96</td>
<td>2.97</td>
<td>0.49</td>
<td>0.49</td>
<td>0.21</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>ACBM</td>
<td>99.01</td>
<td>98.80</td>
<td>7.62</td>
<td>0.49</td>
<td>0.49</td>
<td>10.33</td>
<td>0.41</td>
</tr>
</tbody>
</table>

has the shortest MEW (i.e. expect width) with reasonably high coverage probability. Finally, for moderate to large Poisson rates, all methods except CPBM can be considered as robust method. In these cases, AWM is generally preferable to WBM and GSM since it possesses shorter MEW. Again, ACBM produces the shortest MEW in small confidence levels. All methods have symmetric left and right tail error rates.

REFERENCES


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